Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY
Winter Examination-2015

Subject Name Classical Mechanics

Subject Code :5SC03CME1 **Branch :M.Sc. (Mathematics)**

Semester :3 **Date :12/12/2015** **Time : 2:30 To 5:30** **Marks :70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Define Holonomic constraint. **(02)**

b. Define virtual work. **(02)**

c. Write equation of kinetic energy for a particle moving in plane. **(02)**

d. Define degree of freedom. **(01)**

Q-2 **Attempt all questions** **(14)**

a) Derive Lagrange's equation of motion using D'Alembert's Principal. **(07)**

b) Obtain Lagrange's equation of motion for simple pendulum. **(07)**

OR

Q-2 **Attempt all questions** **(14)**

a) Derive condition for $J = \int_{x_1}^{x_2} f(y, \dot{y}, x)\, dx$ to be extremum. **(07)**

b) Obtain Lagrange's equation of motion for Atwood machine. **(07)**

Q-3 **Attempt all questions** **(14)**

a) Using techniques of calculus of variations obtain the curve between two points (x_1, y_1) and (x_2, y_2) such that the distance is minimum. **(05)**

b) Define cyclic coordinates. If the Lagrangian of a system contains a cyclic coordinate q_j then show that the conjugate momentum p_j is conserved. **(05)**

c) State and prove D'Alemebert's principle. **(04)**

OR

Q-3 **a)** Discuss conservation of linear momentum using Lagrangian formulation. **(05)**

b) State Lagrange's equation of motion in general form. Discuss the case when frictional forces are present. **(05)**

c) Derive Lagrange's equation of motion from Hamilton's principal. **(04)**

SECTION – II

Q-4 **Attempt the Following questions.** **(07)**

a. Write matrix form of Hamilton's equation of motion. **(02)**

b. Determine canonical transformation generated by $F_3 = p_j Q_j$. **(02)**

c. Define Poisson's bracket. **(02)**

d. If M is sympletic then M^{-1} is also sympletic. State whether it is True or False? **(01)**

Q-5 **Attempt all questions** **(14)**

a) Derivation of Hamilton's equation of motion using Legendre's transformation. **(06)**

b) If $L = a\dot{x}^2 + \frac{b\dot{y}}{x} + c\,\dot{x}\dot{y} + f y^2 \dot{x}\dot{z} + g\,\dot{y} - k\sqrt{x^2 + y^2}$ then obtain Hamiltonian. **(04)**

c) Derive condition for canonical transformation when generating function $F = F_1(q, Q, t)$. **(04)**

OR

Q-5 **a)** Derive Hamilton equation of motion from Hamilton's modified principal. **(06)**

b) Discuss conservation of total energy usingLagrangian formulation. **(04)**

c) Show that the transformation $Q = \log(1 + \sqrt{q}\cos p)$, $P = 2\,(1 + \sqrt{q}\cos p)\,\sqrt{q}\sin p$ is canonical. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Derive principle of least action. **(06)**

b) Using condition $M' J M = J$, show that the transformation $Q_1 = q_1$, $P_1 = p_1 - 2p_2, Q_2 = p_2, P_2 = -2q_1 - q_2$ is canonical. **(04)**

c) Prove that fundamental Poisson brackets are invariant under a canonical transformation. **(04)**

OR

Q-6 **Attempt all Questions**

a) Derive sympletic condition for canonical transformation. **(06)**

b) In usual notation prove that $[u,[v,w]]_{q,p} + [v,[w,u]]_{q,p} + [w,[u,v]]_{q,p} = 0$. **(04)**

c) Discuss Routhin's process. **(04)**